



# A topological analysis of the Italian electric power grid

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## Abstract

Large-scale blackouts are an intrinsic drawback of electric power transmission grids. Here we analyze the structural vulnerability of the Italian GRTN power grid by using a model for cascading failures recently proposed in Crucitti et al. (Phys. Rev. E 69 (2004)).

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It is a well-known empirical evidence that electric power grids are inherently prone to frequent disturbances of different sizes [2]. Although most of the disturbances are so small that they affect only a few areas of a city and are unnoticed by the rest of the world, in the last 15 years we have witnessed an increasing number of large-scale blackouts. Large-scale blackouts are due to the concurrent malfunction of a large number of transmission lines and power generators often triggered by the initial failure of a single component of the grid, as the breakdown of a power line. When a power line goes down, the electric current is shifted to the neighboring lines, which in most of the cases are able to handle the extra load. It may happen that some of these lines become overloaded and must redistribute their increased load to their neighbors, eventually leading to a cascade of failures with consequences on the whole electric system. This is exactly what happened in August 10, 1996, when a 1300-MW electrical line

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in southern Oregon sagged in the summer heat triggering a cascading effect that affected 4 million people in 11 different Western states, or in August 14 2003, when an initial disturbance in Ohio caused the largest blackout in the US's history, affecting millions of people for as long as 15 h. Similarly, in September 28, 2003, a failure due to the sag of the electrical line of Lucomagno (Switzerland), caused a blackout for almost the entire Italy and for part of the Southern Switzerland [3].

Given the crucial role that electric power grids have in our society, it has become extremely important to understand the complex phenomena that give rise to a blackout. Researchers have followed different approaches. In particular, simulations that take into account all the electromagnetic processes, and the automation/control layer governing the grid, are extremely difficult and can be performed only on a reduced scale. Conversely, to understand blackouts we are interested in the behavior of the electric power grid as a whole. Moreover such simulations are slow to run, while it would be very important to have an idea in real time of how the system reacts to a given external perturbation.

In this paper we follow a different approach based on the recent development of the science of complex networks [4]: we neglect the details of the electromagnetic processes and we focus only on the topological properties of the grid. Our aim is to demonstrate that the structure of an electric power grid tells us important information on the vulnerability of the system under cascading failures. Recently, a large interest has been devoted to the study of the effects of errors and attacks both on computer generated topologies and on real-world (social, biological and technological) networks [5–9]. In particular, the authors of Ref. [10] have studied the ability of the North American electric power grid to transfer power between generators and consumers when a percentage of the nodes of the networks, representing the transmission substations, are disrupted. The results indicate that the structure of the system is extremely vulnerable to the removal of the nodes with highest load: in fact if the 4% of the nodes with highest load are disrupted all at once, the system experiences a drop of 60% of its performance. Since blackouts in electric power grids are triggered by the initial accidental breakdown of a single substation, in Ref. [1] we have proposed a simple model of cascading failures based on the dynamical redistribution of the load on the networks. In this paper we apply the model to the Italian electric power grid.

We start by briefly reviewing the model for cascading failures. All the details can be found in Ref. [1]. The electric power grid is represented as a graph, in which the  $N$  nodes are the substations (of two kinds: generators or distribution substations) and the  $K$  edges are the transmission lines. To each edge between nodes  $i$  and  $j$  is associated a number  $e_{ij}$  in the range  $[0, 1]$  measuring how efficiently nodes  $i$  and  $j$  communicate through the direct connection. For instance  $e_{ij} = 1$  means that the arc between  $i$  and  $j$  is perfectly working, while  $e_{ij} = 0$  indicate that there is no direct connection between nodes  $i$  and  $j$ .

The efficiency  $\varepsilon_{ij}$  of a path connecting two nodes  $i$  and  $j$  is defined as the harmonic composition of the efficiencies  $e_{lm}$  of the edges constituting the path [1]. The model assumes that each generator sends power to all the other distribution substations using the most efficient paths, i.e., the paths with highest  $\varepsilon_{ij}$ . Under this assumption we determine the best paths for all the possible couples of generators and distribution

substations and calculate the betweenness [11], or *load*, of each node, i.e., the number of best paths that pass through the node. An important ingredient of the model is to attribute to each node a given capacity, i.e., a maximum amount of current that a node can tolerate. The capacity  $C_i$  of node  $i$  is assumed to be directly proportional to the initial load carried by  $i$  [8]:  $C_i = \alpha \cdot L_i(0)$   $i = 1, 2, \dots, N$ , where  $\alpha \geq 1$  is a tolerance parameter of the network and  $L_i(0)$  is the load handled by the node  $i$  at time 0. This is a realistic assumption because the capacity is limited by cost: it would be a waste to build very large and functional nodes if they do not have to carry a high load.

Initially the system is in a stationary state with the edges' weights all equal to 1. The cascade is triggered by the removal of one node that changes the best paths and then also the load distribution. If the load in a node  $i$  exceeds its capacity, the communication involving that node will be degraded and we represent this effect by varying all the edges' weights as follows:

$$e_{ij}(t+1) = \begin{cases} e_{ij}(0) \frac{C_i}{L_i(t)} & \text{if } L_i(t) > C_i, \\ e_{ij}(0) & \text{if } L_i(t) \leq C_i, \end{cases} \quad (1)$$

where  $j$  extends to all the first neighbors of  $i$ .

The update of the edges' weights can cause again the best paths to change, leading to a new redistribution of the load and then to a new update of the edges' weights. In the case the overload to be reabsorbed is large enough, the efficiency degradation will propagate over the whole system in a cascading effect. To evaluate the performance of the network before, during and after the dynamical evolution described by (1), we use the *global efficiency*  $E$ , a measure introduced in Ref. [12] and here defined as the average of the efficiency  $\varepsilon_{ij}$  of the best paths between all couples of generators and distribution substations.

We have built the network of the GRTN Italian electric power grid from the data on the 220 and 380 kV transmission lines of the GRTN web-site [13]. The network has 341 nodes (substations) and 517 edges (transmission lines). As in Ref. [10], we consider different kinds of nodes. In particular we distinguish the *generators*, that are the source of power, from the other *distribution substations* that receive power and transmit it to other substations or distribute it in local distribution grids. In particular we have 102 generators and 239 distribution substations. In Fig. 1a we plot the cumulative node degree distribution  $P_{cum}(k) = \sum_{k' > k} P(k')$  of the GRTN network. The degree  $k$  of a node is the number of its first neighbors [11]. As in most of the networks in which the edges are not costless [14], there is not a particular node or group of nodes that prevail over the others and the distribution can be fitted by an exponential  $P_{cum}(k) = 2.5 \exp^{-0.55k}$ . Under the assumption that each generator has to provide power supply to all the distribution substations through the most efficient paths, we have calculated the load at each node and plotted the cumulative load distribution  $P_{cum}(L) = \sum_{L' > L} P(L')$  in Fig. 1b. Such a distribution can be fitted by a power law  $P_{cum}(L) \propto (785 + L)^{-1.44}$ . This means that, although very homogeneous in the node degree, the network shows a high heterogeneity in the node load: most of the nodes handle a small load, but there are a few nodes that have to carry an extremely high load. It is the failure of the latter that, as we will show in the following, can trigger large scale blackouts. Similar results

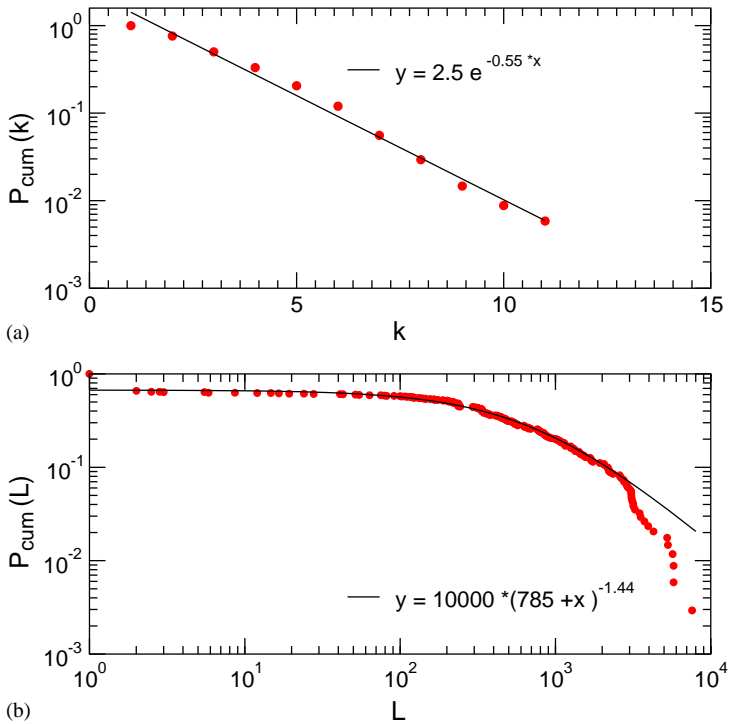


Fig. 1. Cumulative degree and load distributions of the Italian GRTN power grid.

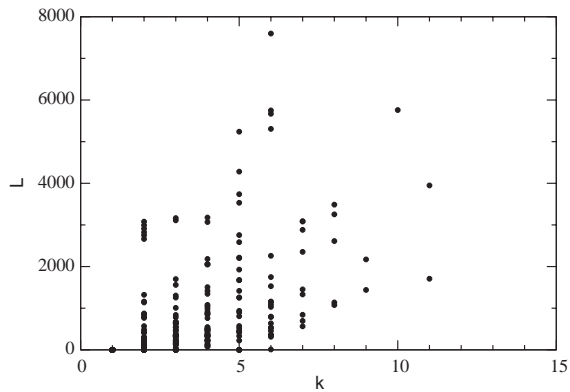


Fig. 2. Plot of degree  $k$  and load  $L$  for each node of the GRTN network.

have been obtained for the Western US [1] and the North American power grid [10]. In Fig. 2 we show that the two quantities, degree  $k$  and load  $L$  are not perfectly correlated in the GRTN network: the fact that some nodes are highly connected does not mean that they are involved in a high number of paths. For instance the node of Pallanzeno, in the north of Italy, is one of the two nodes with the highest degree ( $k = 11$ ), but

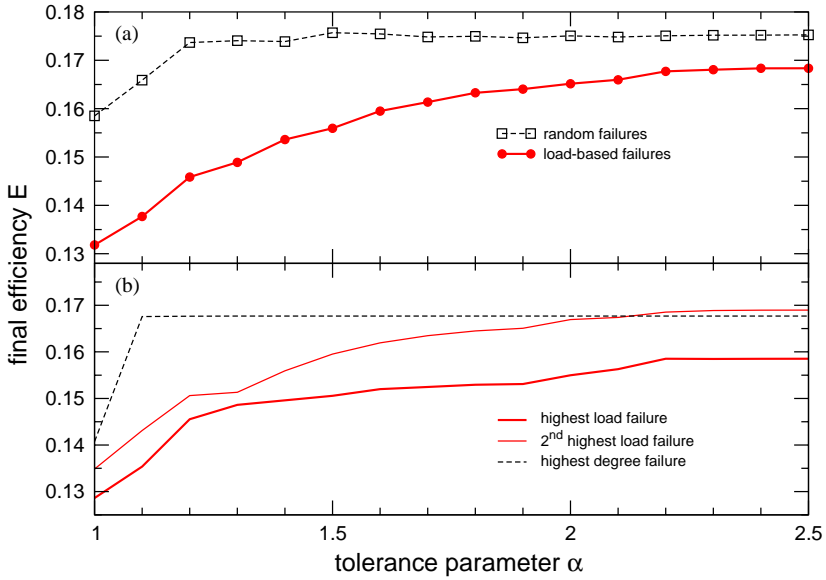


Fig. 3. Final efficiency as a function of the tolerance parameter for cascading failures induced by random and load-based removals. See text for the details.

it carries a load that is less than a fifth of the highest load in the network. Finally in Fig. 3, we show the results of the simulations of Eqs. (1) on the topology of the GRTN. We plot the final efficiency, i.e., the efficiency of the system after it has relaxed to a stationary state, as a function of the tolerance parameter  $\alpha$ . In panel (a) we consider two different situations: random failures and load-based failures. In the first case, we choose at random the initial node to remove (the curve is an average over 50 different choices), while in the second case, we remove a node with the highest load (the curve is obtained by repeating 5 times the simulation for the 5 nodes with the highest loads). The high heterogeneity in the node load leads to a situation in which load-based removals show a very different behavior with respect to random removals. In fact, there is a large region in the tolerance parameter,  $1.2 \leq \alpha \leq 1.5$ , where the network efficiency is unaffected by random failures but suffers the consequences of load-based removals. If the parameter  $\alpha$  is in this region, in most of the cases the overload due to the failure is perfectly reabsorbed by the system. Nevertheless there is a finite, though small, probability that the failure occurs in a node with high load, triggering a cascade mechanism, whose effects propagate over the whole system. In panel (b) we plot three cases obtained when the initial failure occurs: (1) at Suvereto, the node with highest load; (2) at La Spezia, the substation that has the second highest load; (3) at Pallanzeno, the substation with the highest degree but not very high load. Except for very low values of the tolerance parameter, no substantial differences are present among case 3 and the random failures of panel (a) meaning that the node degree is not the key quantity for cascading effects considered in our model.

In conclusion, in this paper we have shown how the topology of an electrical power grid may be relevant to understand the tolerance of the system against failures. In

particular the GRTN network, because of its heterogeneity in the node load, has turned out to be fairly robust to most failures, but very vulnerable to those failures that occur on the nodes with highest betweenness.

Of course, the model shown here, is a heavy simplification of what happens in a real electric power grid which lives at every moment on a delicate balancing act between the amount of electricity requested by the users and the electricity being generated. Moreover, it is not true that each generator has to provide power supply to all the distribution substations. One could think that each generator transfers power only to the nearest distribution substations, but this assumption would be false, as well. It was the situation of the early days of electricity [15], but nowadays the produced power is often redirected hundreds of kilometers away. Besides electricity does not always follow the most efficient paths and the existence of a line between two substations does not always imply that power can be transferred across it as there may be other constraints present [10]. Further complicating matter, electricity flows through the grid primarily as alternating current. So alternating current frequencies at each substation must be offset in a precise way to keep power flowing in the right direction.

The assumptions of our model makes it too simple for the real world. A valid and complete modeling aiming at understanding and improving power transmission infrastructures should take into consideration a more complicated system composed by a physical layer, an automation and control layer, a supervision and management layer and a strategic and politic layer [16]. However, the aim of this study was to show that the topology plays a fundamental role in the characterization of power transmission systems and that it is not possible to leave it aside in any kind of modeling.

## References

- [1] P. Crucitti, V. Latora, M. Marchiori, cond-mat/0309141, and *Phys. Rev. E* 69, 2004, in press.
- [2] B.A. Carreras, D.E. Newman, I. Dolrou, A.B. Poole, *Proceedings of Hawaii International Conference on System Sciences*, 2000.  
M.L. Sachtjen, B.A. Carreras, V.E. Lynch, *Phys. Rev. E* 61 (2000) 4877.
- [3] R. Bacher, U. Naf, *Rapporto sul blackout italiano del 28 Settembre 2003*.
- [4] S.H. Strogatz, *Nature* 410 (2001) 268;  
R. Albert, A.-L. Barabási, *Rev. Mod. Phys.* 74 (2002) 44;  
S.N. Dorogovtsev, J.F.F. Mendes, *Evolution of Networks*, Oxford University Press, Oxford, 2003.
- [5] R. Albert, H. Jeong, A.-L. Barabási, *Nature* 406 (2000) 378.
- [6] P. Crucitti, V. Latora, M. Marchiori, A. Rapisarda, *Physica A* 320 (2003) 622.
- [7] P. Holme, B.J. Kim, *Phys. Rev. E* 65 (2002) 066109.
- [8] A.E. Motter, Y. Lai, *Phys. Rev. E* 66 (2002) 065102(R).
- [9] Y. Moreno, R. Pastor-Satorras, A. Vázquez, A. Vespignani, *Europhys. Lett.* 62 (2003) 292.
- [10] R. Albert, I. Albert, G.L. Nakarado, *Phys. Rev. E* 69 (2004), in press.
- [11] S. Wasserman, K. Faust, *Social Networks Analysis*, Cambridge University Press, Cambridge, 1994.
- [12] V. Latora, M. Marchiori, *Phys. Rev. Lett.* 87 (2001) 198701.
- [13] GRTN S.p.A. Cartografia rete di trasmissione, <http://www.grtn.it>.
- [14] L.A.N. Amaral, A. Scala, M. Barthélémy, H.E. Stanley, *Proc. Nat. Acad. Sci. USA* 97 (2000) 11149.
- [15] A.-L. Barabási, *The New York Times*, August 16 2003.
- [16] C. Balduccelli, S. Bologna, *Proceedings of the Ninth Annual Conference of the International Emergency Management Society*, 2002.