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Topological properties of high-voltage electrical transmission networks

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Abstract

The topological properties of high-voltage electrical power transmission networks in several UE countries (the Italian 380 kV, the French 400 kV and the Spanish 400 kV networks) have been studied from available data. An assessment of the vulnerability of the networks has been given by measuring the level of damage introduced by a controlled removal of links. Topological studies could be useful to make vulnerability assessment and to design specific action to reduce topological weaknesses.

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Keywords: Electrical transmission networks; Vulnerability; Topology

1. Introduction

Recent electrical power outages experienced in the north-east area of US and in Canada (August 14, 2003) and on a large portion of the italian country (September 28, 2003) have strongly pushed forward the problem of the analysis, the control and the fast healing of complex critical infrastructures, after the occurrence of some type of fault. The analysis issued after these events [1,2] have highlighted some key points deserving a careful consideration:

- complex networks, such as power transmission lines, are vital structures for all countries. Although being, in general, reliable and fault tolerant, they might be prone to weaknesses from the topological up to the electrical levels; failures of their functions, albeit rare, might engender severe problems originating considerable risks and costs;
- there is still a need of instruments for the on-line analysis of networks functionality which may rapidly provide decision support for healing (or self-healing) actions; complexity, in large networks, originates non-linear and non-local effects which must be treated with suitable approaches which consider the network at large;
- there is a very high level of interconnection between critical infrastructures whose consequences cannot be easily predicted on the basis of simple models; tight inter-dependencies between networks are at the origin of cascading failures (the failure of one induces malfunctions into the others which, in turn, feed back, with further events, the network where failures initiated etc.). This is a further origin of network's vulnerability which should attract the necessary attention from both the theoretical and the technological sides.

Although the solution of the problems issued by these points is far to be simple and achieved, several novel theoretical advancements might help in sheding a new light on this matter [3].

It has been noticed that many "real-world" networks arising from non-supervised growth processes display common topological features [4–6]. Networks in biology (metabolic and protein–protein interaction networks), in sociology (scientific publication co-authors network), in communication engineering (the Internet), in the information society (the web pages), although describing different systems and phenomena, show similarities in their structure [5]. The most striking among them is the emergence of a network topology with a distribution of the node's degree k, $P(k)$, characterized by a power-law function of the type [4–6]:

$$P(k) \sim k^{-\gamma} \tag{1}$$

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with the value of γ restricted in a narrow interval ($2 < \gamma < 3$) [5]. The structure of the networks originating power-law distribution of node's degree is "scale-free" [3,4], where highly (hubs) and loosely (leaves) connected nodes coexist. These exhibit a number of interesting functional properties which have been widely discussed [5]. Although diverse types of networks display a scale-free topology, many others cannot be structured that way, due to some geometrical constraint [7]. Self-organized networks with a scale-free topology, due to their relevance for allowing the understanding of the origin of their "common" organization and the presence of an underlying growth mechanism, have attracted, in recent years, much more interest than exponential or random networks. However, the advancements made in the topological and the spectral analysis of these systems can be easily transferred to other specifics, to helping the description and the analysis of objects such as the electrical power networks with a single-scale structure. These have a more regular structure, characterized by low values of the node's degree and showing a cumulative degree distribution with exponential or gaussian-type decay [3,7].

The intent of the present work is to present methods and findings which, disregarding the effects related to the specific type of load on the network, is able to detect some vulnerability at the topological level. There have been a quite large effort to describe and analyze the effects of nodes and lines deletion in a realistic transmission line. Some authors have simulated the effects, at the level of load shedding, resulting from the indertiction of some resources [23]; others have superimposed, to a generic network, an heterogeneous distribution of loads able to engender, upon disabling elements of the networks, unpredictable cascade events [9–11]. Our work considers, in a more general way, the situation of a network described at the level of a connected, non-oriented graph: disregarding the kind of load acting on it, we concentrate on the relations between topology and robustness.

Data have been taken from a coarse-grain description, under the form of network's graphs, displayed in the web sites of the national agencies operating the electric transmission lines in the different countries [12]. Maps have been transformed into graphs from which adjacency matrices have been deduced. In the italian case, the availability of official documents containing the description of the whole electrical transmission network (comprising lines from 380 kV down to 120 kV) allowed to make a deeper assessment of the properties of a more realistic network, not limited to a single voltage. The analysis of only the highest voltage network, however, should not be regarded as a weakness of the present study. It has to be reminded that the highest voltage networks, although being composed by a small number of nodes and links, are the most important components of the transmission networks which are totally ineffective in absence of the correct functioning of the highest voltage component. Several properties of the networks have been evaluated from these data: beside the description of the general topological features (average degree, degree's distribution and its cumulative distribution function), we have performed a detailed analysis of properties such as the distribution of the node's distance and, by a suitable application of the spectral analysis, of properties related to networks vulnerability. At the end, we will attempt to sketch a number of actions to be taken to reduce the impact of network's weaknesses on their vulnerability. These actions might be viewed as a possible way in which the "complex systems view" of the electrical power networks might be useful for helping the design of a new generation of tools for the analysis of complex systems and for supporting the (human or cyber) operator in the control of critical infrastructures.

2. Definition and evaluation of network's properties

Data have been taken from [12], where they are stored under the form of pictures representing the national maps, displaying the location of the nodes and links of their high-voltage networks. In these maps, nodes are power sources (thermal and hydroelectric plants) and substations, links represent transmission lines of two types: single- and double-circuit lines. Raw data have been suitably transformed into a mathematical description of the networks, each identified by a set of data, GI, GF, GS and GI2 (the I-380 kV, F-400 kV, S-400 kV networks, and the fine-grain italian network comprising lines from 380 kV down to 120 kV, respectively) where $G_i = (N_i, L_i)$ (N_i and L_i, with i = I, F, S, I2 are the relative number of nodes and links). The sizes of these networks are reported in the first two columns of Table 1.

One of the mathematical objects allowing a complete definition of the network topology is the Adjacency matrix A; if the network has undirected and unitary links, assumed to hold hereafter, it is defined as $A_{ij} = 1$ if nodes i and j are connected, 0 otherwise. All the relevant properties of the network can be deduced from the analysis of the Adjacency matrix [3,13]. A further matrix which can be associated to the network, which will be used in this work, is the so-called Laplacian Matrix L, defined as $L = D - A$ (where D is the diagonal matrix having $D_{ii} = k_i$, with k_i being the degree of the ith node). Further insights on

Table 1
General properties of the analyzed networks

Network	N	L	$\langle k \rangle$	c	k_{max}	d	$\langle l \rangle$	E	n_l
G_I (I 380 kV)	127	171	2.69	0.156	7	25	8.47	0.173	3
G_F (F 400 kV)	146	223	3.05	0.279	8	15	6.61	0.197	7
G_S (S 400 kV)	98	175	3.57	0.316	9	11	4.92	0.259	7
G_{I2} (I fine grain)	1926	2240	2.33	0.019	15	42	14.78	0.082	9

N is the number of nodes, L the number of links, $\langle k \rangle$ the average degree; c the average clustering coefficient; k_{max} the degree of the node with the maximum number of connections (hub); d is the network diameter, $\langle l \rangle$ the characteristic path length; E is the network's efficiency [13]; n_l the number of links connecting the two sub-networks solution of the "min-cut" problem.

the structure and the properties of the networks can be gained by the spectrum analysis of the A and/or the L matrices [5,14]. Topological quantities are determined to gain a first insight on the network's static properties. We have evaluated the clustering coefficient c [4,15] (the average value of the fraction of links present between the first neighbors of a node with respect to the maximum possible number of links between them); the network diameter d, defined as the maximum point-to-point distance in the network; the characteristic path length $\langle l \rangle$, which is the averaged value of the point-to-point distance over the $N(N-1)/2$ distances. We have, moreover, evaluated the network efficiency E, defined as in [16]

$$E = \frac{1}{N(N-1)} \sum_{ij} \frac{1}{d_{ij}} \tag{2}$$

where d_{ij} is the distance between nodes i and j. The quantity E generalizes the concept of "small world" [4,5] for any network, regardless of its topology.

We have also evaluated some property deduced by the spectral analysis. There is a wide literature on the spectral analysis of the A and L matrices associated to a graph [15,17,18]. As in a previous attempt, made to analyze the network of the Internet AS-level routers [19], we focus on a specific result derived by the spectral analysis of the L matrix and the so-called "min-cut" theorem [20–22]. This can be stated as follows. The lowest eigenvalue λ_1 of the L matrix is always vanishing ($\lambda_1 = 0$) and the orthonormalized components of its associated eigenvector $v(\lambda_1)$ are all equal to $1/\sqrt{N}$ (N being the number of nodes). The components of the second eigenvector $v(\lambda_2)$ associated to the second eigenvalue λ_2 of the Laplacian ($\lambda_2 \neq 0$) have, in turn, different signs. The "min-cut" theorem provides a recipe which, starting from the analysis of the sign of the components of the second eigenvector $v(\lambda_2)$ of L, allows the partition of the network into two nearly equal sub-networks connected via the minimum number of connections. The first sub-network is formed by nodes with a positive component, the second by those with a negative component. The "min-cut" theorem ensures that the number of links to be cut n_l, between these two set is the smallest possible cut between two nearly equal subset of the whole graph, i.e. the cut has a minimum "weight". We have thus defined n_l as the number of links joining nodes belonging to the different sub-networks (i.e. from the total number of links L, we count only those joining nodes belonging to different sub-networks). The links defined by the "min-cut" algorithm are those whose failure would induce the "most effective" perturbation to the network by producing the maximum number of "effective broken links". The values of n_l, resulting from the spectral analysis of the network's Laplacians are reported in the last column of Table 1.

Although for the specific case of electrical power network the resulting cut does not necessarily imply the physical islanding of some region of the net, for other networks (such as the internet or the web) the cut individuated by the suggested analysis does effectively correspond to a true disconnection of resources. On the other side, deliberate islanding is known to be a fruitful strategy to be adopted to limit the spread of cascade disturbances [23]. The proposed method could be thus suggested to define a number of sections usable for a rapid deployment of that strategy.

Data reported in Table 1 provides a picture of the coarse-grain electric transmission lines as sparse networks (low ratio $2L/[N(N-1)]$) with a low average degree ($\langle k \rangle$ 3.6), a quite low value of the maximum degree (which ranges between 7 and 9) but with a quite large clustering ($0.156 < c < 0.310$). The networks, moreover, display characteristic path lengths (between 5 and 8.5) larger than those corresponding to random networks ($\langle l_{\text{rand}} \rangle = \log N / \log \langle k \rangle$), with maximal values as large as 25 (in the I-380 kV case). These parameters are neither the expression of a random network, nor of a scale-free one. In fact, large clustering values are not typical of scale-free networks grown by preferential attachment (special growth mechanisms have to be employed to significantly enhance the c parameter [24]). A further indication on the relative difference of the GI network with respect to GF and GS is constituted by the value of n_l. The values of this quantity for the GF and GS networks are consistent with those pertaining to a scale-free network of same size and similar value of c. The latter parameter is relevant to detrmine the extent of n_l in a scale-free network; in fact, a scale-free network of the same size and clustering of GI would have displayed a value of $n_l \approx 12$; this value should have been even larger $n_l \approx 14$) for a pure random network (with a lower c value). These data shows a substantial equivalence between the three networks (GI, GF and GS), pointing however to some specificity of the GI network, mainly revealed by the large d value and the very low extent n_l of the min-cut. A specific interest for the network's classification is assumed by the distribution and the cumulative distributions of the specific topological quantities. We have firstly evaluated the degree's distribution and its cumulative distribution to compare data from the considered networks with those coming from the inspection of transmission lines in other countries recently reported [3]. This work, dealing with the US electrical power network, has pointed out the specific exponential shape of the cumulative distribution of node's degree. The cumulative distribution $P(k > K)$ is defined as the probability of having a node whose degree is higher than a given value of K. Our estimates for the four networks, P_{I}, P_{F}, P_{S} and P_{I2} (displayed in Fig. 1) have allowed to find

$$P_{\text{I}}(k > K) = \exp(-0.18K^2) \tag{3}$$

$$P_{\text{F}}(k > K) = \exp(-0.21K^2) + 0.18\exp[-0.25(K-4.0)^2] \tag{4}$$

$$P_{\text{S}}(k > K) = 0.96\exp(-0.17K^2) + 0.25\exp[-0.19(K-3.9)^2] \tag{5}$$

$$P_{\text{I2}}(k > K) = \exp(-0.32\,K^2) + 0.065\exp[-0.20(K-4.0)^2] \tag{6}$$

which are different from those reported in [3]: $P(k > K)$ has a simple exponential form for the US electrical grid. There are several points, however, which lead the european networks to be somehow different from the US one. The latter, in fact, is huge containing more than 10^4 nodes, with degree values ranging

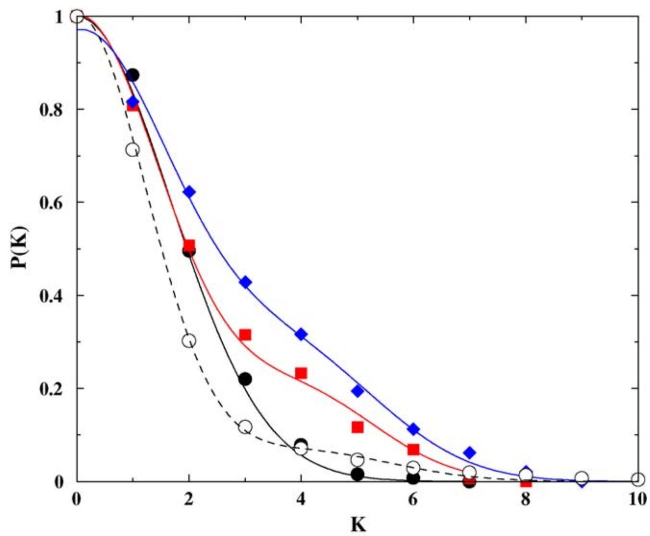

Fig. 1. Cumulative distribution of the node's degree of the GI (full circles), GF (squares) and GS (diamonds) networks: lines correspond to the best fits (see, e.g., Eqs. (3)–(5)). The GI2 network (empty circles) has a best fit represented by Eq. (6).

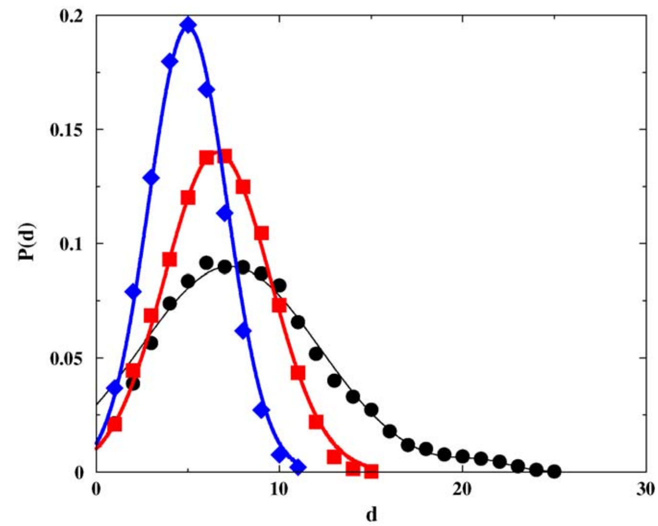

Fig. 2. Distribution of the shortest path lengths between all the nodes of the GI (dots), GF (squares) and GS (diamonds) networks: lines correspond to the best fits obtained by using the following gaussian functions: $P_{\rm I}(d) = 0.09 \exp[-0.021(d - 7.3)^2] + 0.004 \exp[-0.2(d - 21)^2]$, $P_{\rm F}(d) = 0.14 \exp[-0.06(d - 6.61)^2]$, $P_{\rm S}(d) = 0.195 \exp[-0.11(d - 5)^2]$.

up to 25. The considered networks are much smaller, with the maximum degree value smaller than 10. The low value of the highest degree is such to produce a gaussian-type decay of the cumulative distribution of the degree. According to recent studies [7], gaussian type behavior for the decay of the cumulative distribution of node's degree is to be expected when the growth is characterized by the local onset of a "cost" for adding new links to existing nodes. When some node has reached a threshold degree, it might become "inactive" and its degree cannot grow any longer [7]. The constraints might even induce the network to develop a more-than-exponential decay, such as a gaussian tail; in [7], gaussian decay for the cumulative distribution of node's degree has been reported for a regional (Southern California) electrical network.

We have evaluated the distribution of the lengths of the shortest paths, whose shape is reported in Fig. 2.

It is interesting to notice how the peculiar geography of the country is also reflected into these data. In Fig. 2, for instance, the distribution of the node's distance in GI is bi-modal, with the two best-fit gaussians being centered at $d_1 = 7.3$ and $d_2 = 21$. This is still related to the form of the italian country; the low d_1 value averages the distances between nodes belonging to the same geographical district, the large d_2 value, in turn, is the average distance between nodes belonging to different districts. The scarce number of shortcuts between North and South compels the use of the few longitudinal links and that of a serial connection, thus resulting in large internode distances.

We have attempted to give a major emphasis to the issues of networks vulnerability. We have associated the concept of network vulnerability to a number of different quantities: the extent of critical sections present in the network and the conditional probability of disconnecting nodes given the removal of a number of links [25]. We have defined critical sections the set of links associated to a network partition made on the basis of the min-cut procedure. As it has been previously explained, the min-cut procedure is able to divide the network into two sub-networks whose interconnection has the minimum weight (i.e. is composed by the minimum number of links n_l). Lower the value of n_l, higher the vulnerability of the network (when the links of a given critical section are removed, the two sub-networks resulting from the cut are totally disconnected). If we apply the procedure to the Italian GI network, it results to be divided in two nearly equal sub-networks, the first containing 51 nodes, the second 76 nodes. The number of links connecting these two halves is $n_l = 3$; the position of the related links (whose set will be referred to as first-level critical section) in the network is reported in Fig. 3 (thick black links). If we take the resulting two sub-networks and we repeat the min-cut procedure on each of them, we will identify two second-level critical sections; the relative links are reported in Fig. 3 in thick blue lines. It is interesting to notice that both first- and second-level critical sections have comparable extent (the values of n_l for the second-level critical sections are equal to 3 and 4, see Fig. 3). This result reflects a multi-scale vulnerability of the considered network. In fact, the vulnerability of the network (low n_l value) extends to different sections of different sizes.

Low values of n_l of first-level critical sections have been also reported for early stages of growth of the US AS-level internet routers [19]; however, later stages of growth of that network have been found to be characterized by a much larger value of n_l, thus indicating a very rapid increase of this parameter (its growth has been estimated to be 20 times faster than the corresponding growth of the number of nodes). The growth mechanism which has been proven to be consistent with the resulting figures of the principal networks properties is the so-called Triad formation mechanism [24]. Large n_l values are thus an emerging feature of unconstrained network's growth; this property seems to be associated to some fundamental property needed to ensure a correct functioning of the network. An immediate short-cut could be

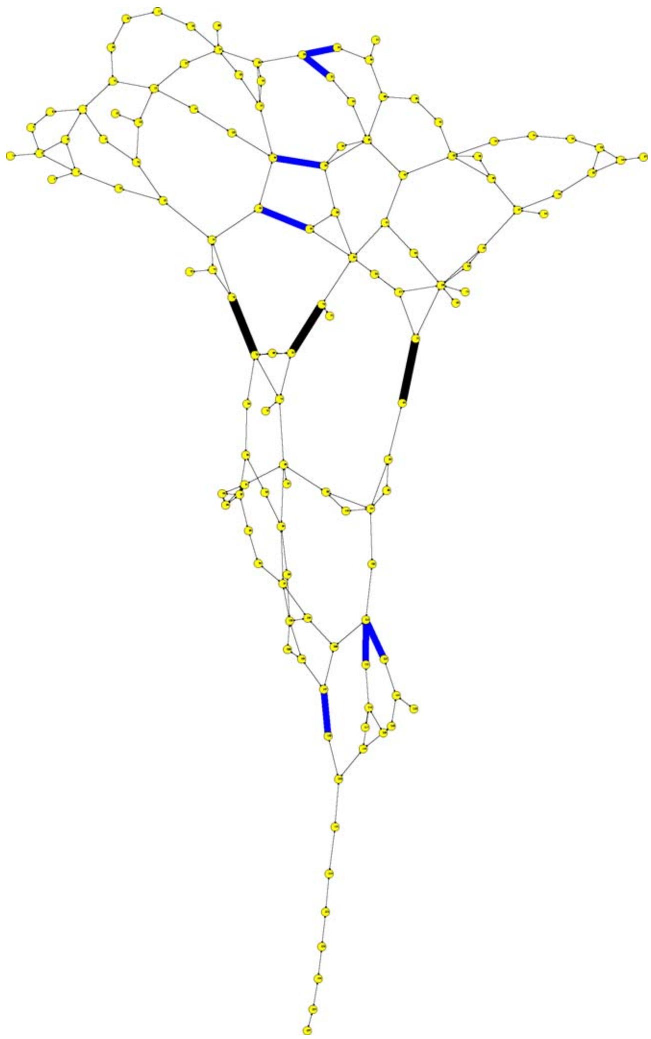

Fig. 3. GI network after two successive applications of the min-cut procedure: thick black links refer to the first critical section resulting from the first iteration; thick blue links refer to the critical sections resulting from the successive application of the min-cut procedure to the two sub-networks defined by the first application of the min-cut.

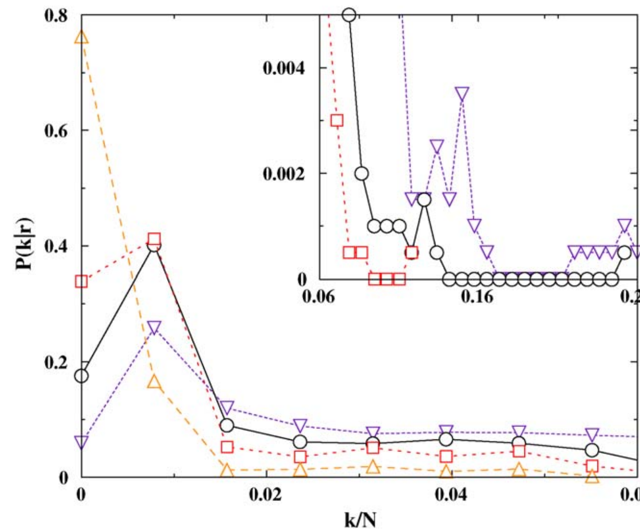

Fig. 4. Conditional probability $P(k|r)$ of having k nodes disconnected given r links removed, in the GI network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa is normalized over the number of nodes. Legenda of symbols: triangle up, $r = 2$; square, $r = 7$; circle, $r = 10$; triangle down, $r = 15$.

envisaged between the n_l value and the network vulnerability. The following computer experiment has been realized to assess the existence of a true functional relation between n_l and that property in the considered electrical transmission networks. The GI, GF and GS networks have been submitted to a controlled links removal, to determine the extent of the damage introduced by this action. The damage resulting from the cut of $1, 2, \ldots r$ links has been defined as the number of nodes which result to be disconnected upon such action. More precisely, we have evaluated the conditional probability $P(k|r)$ that k nodes become unreachable upon the cut of r links of the network. This test has been carried out by suppressing r links and by subsequently recalculating the distance among all the different $i - j$ couples of the networks. If a node cannot be reached by any of the others, that node is defined as "disconnected" and the value which quantifies the extent of the introduced damage is augmented by one. The tests with $r = 1$ have been carried out exhaustively: one at a time, each link has been suppressed and the number of disconnected nodes has been evaluated after each removal. The link has been then reintroduced, a further link suppressed and the procedure iterated. For $r \geq 2$, a sample of 2×10^3 r-uples of links have been randomly selected; the reported results refer to an average over these samples. Figs. 4–6 report the conditional probability $P(k|r)$ in the three networks. This is a robustness index allowing the measure of the extent of the network resistance to faults such as the loss of an r-uple of links. This quantity is a clear indication of the level of vulnerability of the network versus the action of links removal. In the analyzed cases, results point to a substantial vulnerability of the GI network (Fig. 4) as, even in the case of the removal of $r = 2$ links, there is a considerable probability $P(k > 2|2) \sim 5.8\%$ of having $k > 2$ disconnected nodes (in the GF case, $P(k > 2|2) \sim 0.3\%$). In turn, the GF and GS networks show a quite higher robustness as the probability $P(k > 2|7) \leq 3\%$ (to have $k > 2$ disconnected nodes upon removal of $r = 7$ links, see Figs. 5 and 6).

Results in Figs. 4–6 are reported in terms of the fraction of nodes disconnected upon the removal of r links. The normalization of this datum allows a better comparison of the different impact that the fault events have on the networks. Also this property points to a specific weakness of the GI network with respect to the others. Beside the specific form of cumulative distribution of node's degree, also this property seems to be related to the specific geography of the country. As a mere qualitative indication of that, when the network design has to be tightly mapped onto the country morphology and its peculiar distribution of sources and loads, the resulting structure of the network unavoidably exhibits some weakness. In the case of the Italian network, the specific morphology, with a narrow peninsular body, "squeezes" the structure of the network. In the region where the continental, north part of the country becomes a peninsula and the east-west extension of the country reduces to a few hundreds kilometers, the network shows a weakness related to the lack of a robust

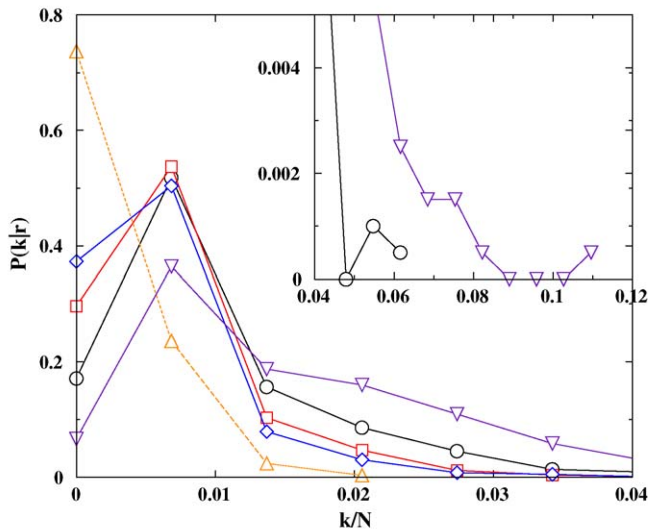

Fig. 5. Conditional probability $P(k|r)$ of having k nodes disconnected given r links removed, in the GF network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa is normalized over the number of nodes. Legenda of symbols: triangle up, $r = 2$; diamond, $r = 6$; square, $r = 7$; circle, $r = 10$; triangle down, $r = 15$.

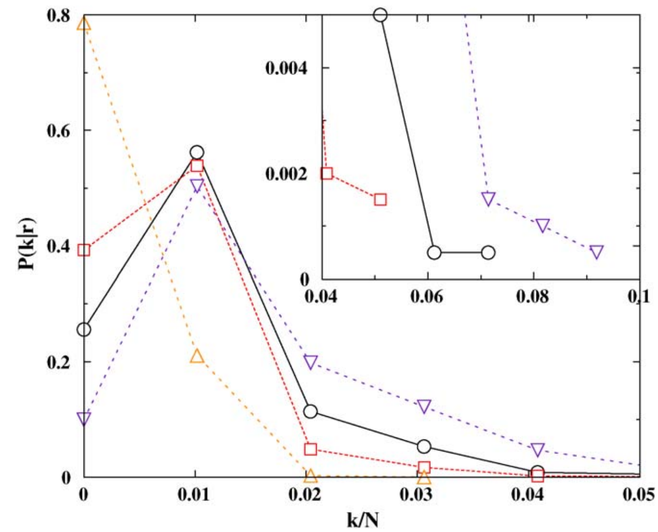

Fig. 6. Conditional probability $P(k|r)$ of having k nodes disconnected given r links removed, in the GS network. The inset reports, on a larger scale, the behavior for high k values. Symbols are valid throught the whole picture. Abscissa are normalized over the number of nodes. Legenda of symbols: triangle up, $r = 2$; square, $r = 7$; circle, $r = 10$; triangle down, $r = 15$.

interconnection between the north and the central-south parts. In fact, the min-cut analysis of the GI network shows that the narrowing of the country (and, consequently, of the network) reduces of the number of links; this fact, in some sense, eases the cut of the networks into two almost equal sections. This division moreover establishes by only three links, thus becoming a severe source of weakness in the network. This effect is almost absent in France and Spain where the countries morphology prevents its occurrence ($n_l = 7$ in both cases). The fact that the GI2 net displays a value of $n_l = 9$ should not mislead: the GI2 graph is constituted by the merger, in a unique graph, of several high-voltage lines, while GI represents just the highest voltage network. The efficiency of this latter, however, determines the functioning of all the other transmission lines. In this respect, GI, GF and GS, as representing equivalent "samples", can be directly compared.

To show a possible relation between vulnerability and the value of n_l, we have attempted to "optimize" the structure of the GI network to increase the value of n_l resulting from the min-cut procedure. This has been attempted by inserting just one extra link in the network and by recording the resulting variation of n_l. and that of the vulnerability, expressed in terms of the variation of the $P(k|r)$ function. The optimization procedure ended up with the following solution: if two specific nodes are connected (one in the centre-north, the other in the centre-south), the number of links solution of the min-cut procedure n_l rises from $n_l = 3$ to $n_l = 12$.

We have also shown that this specific action is also able to decrease the network vulnerability. In fact, if we evaluate the conditional probability $P(k|r)$ in the modified network, we show how the simple introduction of that new link, is able to substantially reducing the vulnerability of the network: in fact, as $r = 15$, there is a low probability to have a number of disconnected links $k > 15$ in the modified network, while in the unperturbed

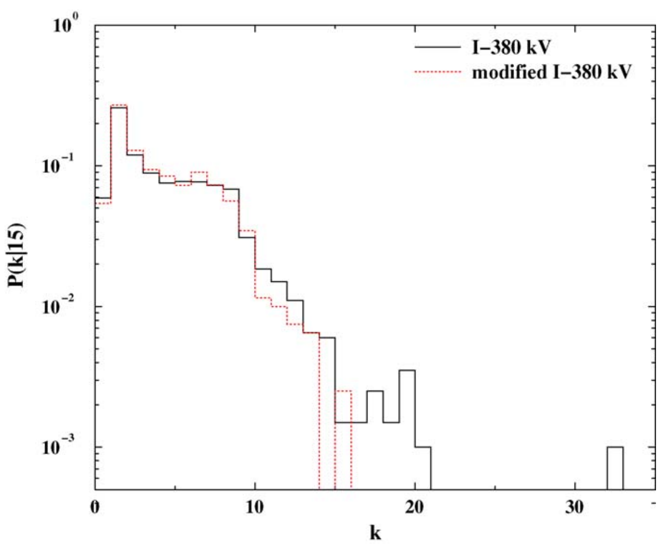

Fig. 7. Linear-log plot of the conditional probability $P(k|15)$ of having k nodes disconnected given $r = 15$ links removed, in the GI network: full line is the original I-380 kV network, dotted line is the same network with a new extra link (which brings to a maximum the number of links solution of the min-cut problem) added.

GI network, there is a non-vanishing probability of disconnecting up to 33 nodes (Fig. 7). The same holds for $r < 15$ even if the effect is less pronounced.

3. Conclusions

This work has highlighted the role that topological analysis might play in assessing several properties of a complex network, particularly those related to its robustness and fault-tolerance. These methods, when combined to spectral analysis, allow to producing an almost complete description of the topological response of the network. A number of other properties might be disclosed by a network analysis where the adjacency matrix is substituted by a load (or capacity) matrix. This can be used to evaluate both static (equilibrium problem) and the dynamic series of events which may be related to the overload of the network's components induced by some faults [9,10] or terroristic attack [8]. This analysis, which will adapt, to the specific problem, a theoretical approach recently proposed [26], will be the object of a further investigation.

This analysis, performed on three european high-voltage transmission networks, has located a number of properties which are common to these networks, independently by their structure, and others which are rather peculiar, somehow reflecting the specific geography of the country. Data shows that the coarse-grain networks display topological properties which are typical of "small world" systems. In fact, they exhibit a very large clustering coefficient (much higher than the corresponding value in random networks $c_{\text{rand}} = \langle k \rangle / N$) and a large characteristic path length, larger than that pertaining to random networks. This fact has been already noticed by other authors who have analyzed the US electrical transmission lines [3]. Data of the GI2 network (seen as the GI at a larger stage of growth) confirm the hypothesis that constrained- growth mechanisms are not able to improve networks properties: clustering decreases and the value of n_l has only a small increase. These growths data should be compared to those resulting from an unconstrained growth, such as that experienced by the US AS-level router [19]. The vulnerability analysis performed on the basis of two indicators (the size of the links comprised in the critical sections and the conditional probability of having nodes disconnected upon links removal) has identified several potential weaknesses in the considered networks. In conclusion, it must be recall, however, that the analysis of the network's topology is able to cope only with a single part of the problem. In fact, sequence-of-events analyses of the largest blackouts experienced in 2003 have shown that, although being triggered by events occurred on the physical network, most of the resulting effects have been a consequence of errors and faults at the organizational and the cyber-control levels [27], made when attempting to heal the effects produced by the initial events. Moreover, there are other major effects, occurring at the functional level (power flow, etc.) which precisely define the overall effect of the failure, even after the occurrence of some critical failure. All these effects have not been accounted for in our simple topological model, which attempts to deal with a single aspect of the problem.

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