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Influence of the topology on the power flux of the Italian high-voltage electrical network

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Abstract. – A model of the Italian 380 kV electrical transmission network has been analyzed under the topological and the behavioural viewpoints. The linear system, resulting from the DC power flow model used to evaluate the power flux has been solved on the basis of input conditions (injected power - extracted power, arc's reactances and the maximum flux capacity of each node) taken from real data. The impact of links removal on the network's structure and function has been also evaluated. The vulnerability of the network under load conditions has been estimated by evaluating the power flux redistribution along the lines subsequent to the removal of a growing number of lines. When the perturbed network cannot sustain a given input–output demand, we have estimated the power which can be sustained by the network to optimize the `Quality of Service`, defined as the difference between the expected and the effective dispatched power. The different lines of the network have been classified according to the amount of power that the network must reduce, to keep alive, upon their removal.

Introduction. – The high-voltage electrical transmission network occupies a relevant place among the Critical Infrastructures (CIs); it transmits the high-power flux from the power plants to the distribution networks which constitute the more diffuse electrical infrastructure. It is, therefore, at the top of a hierarchical view of "electrical CI"; its incorrect functioning has a strong repercussion on the whole transmission and distribution process. Its functioning, moreover, strongly influences that of many other CIs (such as, e.g., the internet and, in general, all communication infrastructures, the railways etc.), and it has a strong impact on the life of citizens (see, in this respect, the assessment of the effects of the blackout experienced by Italy on September 2003 (1)).

In the present work, we study the properties of the italian high-voltage (380 kV) electrical transmission network (HVIET hereafter). Its aim is to estimate the vulnerability of the

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(1)Interim Report of the Investigation Committee on the 28th September 2003 blackout in Italy (http://www.ucte.org/publications/library). UCTE is the "Union for the Co-ordination of Transmission of Electricity" ; it represents the association of transmission system operators in continental Europe, providing a reliable market base by efficient and secure electric "power highways".

network based on the analysis of its topological properties and on the results of a simple model of power transport. A major outcome of the work would be to establish some correlation between the *topological* properties of the different parts of the network and their functional relevance.

Several works (see, among others, [1–3]) have pointed out the existence of some relation between topology and vulnerability. Topological analysis is able to predict points of *structural* vulnerability (i.e. indicating the links whose failure would induce a severe *structural* damage through the physical disconnection of its parts). In the present work, we wish to investigate the vulnerability issue also from the viewpoint of the network's *functioning.* The electrical power flow establishes in a quite complex way along the different lines, depending on topology, on the position in the networks of sources and loads, on the line's characteristics etc. It may happen that lines having topological relevance have, in turn, a bare relevance on the flow distribution. It is therefore important to associate to a vulnerability study based on the simple *topological* analysis of its components, the analysis of the impact that faults (i.e. arcs and/or node's removal) have on the network's functionality.

This work is part of a more general program aimed at studying the interdependencies between CI's [4] and the effects that these dependencies might induce in strongly interconnected networks.

Model and computational method. – We have analyzed data relative to the Italian high-voltage (380 kV) electrical transmission network (HVIET). This study follows previous works performed to unveil, from topological analysis, a number of features of the HVIET [2, 3]. The present work, however, aims at introducing some vulnerability assessment based on a *dynamical* model of the network, by reproducing the power flow conditions and by evaluating the impact on these conditions caused by the introduction of some faults.

Network's data have been inferred from the analysis of the public documentation (Gazzetta Ufficiale).

HVIET can be represented by an undirected graph of N nodes and E lines. Available data allow to attribute to each node the quality of being a *source* node S (where part of the power is inserted in the network), a *load* node L (where part of the power is extracted from the network) and a *junction* node J (which is neither a S nor a L node). The topology of the HVIET is reported in Fig.1(left) where S nodes are green, L nodes are red and J nodes are black circles. HVIET consists of $N = 310$ nodes and $E = 359$ arcs. There are $S = 97$ source nodes, $L = 113$ load nodes and $J = 100$ junction nodes. Some arcs (14 over 359) are constituted by *double* lines (fig.1, left). Points of cut of the network (which is connected with other european networks) have been substituted with "fictitious" source nodes where the same amount of electrical power received by foreigner countries is pumped into the network.

Several topological properties have been analyzed on the HVIET network; among them the distribution $P(k)$ of the node's *degree* k (the *degree* is the number of links connecting each node to its nearest neighbors) which allows to "classify" its topology. The HVIET $P(k)$ is reported in fig.2 [5, 6]. The network has a limited number of hubs, whose maximum degree is $k_{max} = 10$. $P(k)$ and the cumulative degree distribution $P(k > K)$ are both likely to be fit to an exponential (single–scale network [1, 2, 7]). The latter distribution can be fitted to $P(k > K) \sim e^{-0.55K}$ in agreement with previous findings for the North-american power grid [1]. A further property measured on the HVIET network is the average *clustering* coefficient C [2] which results to be as small as $C = 2.06 \cdot 10^{-2}$.

We have also evaluated further topological quantities such as the *betweenneess* centrality,

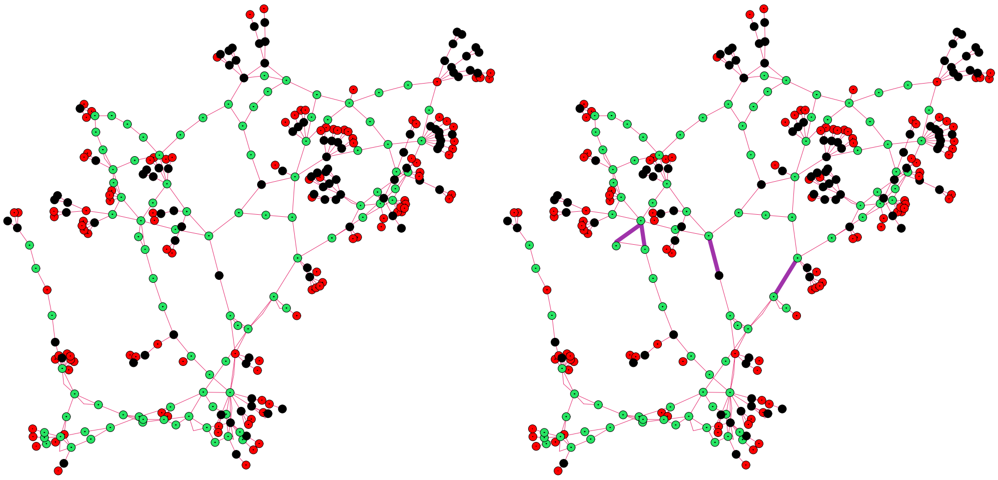

Fig. 1 – Left: the Italian high-voltage (380 kV) transmission network resulting from the available data. S nodes are green, L nodes red and J nodes are black circles. Right: same graph with highlighted links whose removal allows to bisecate the graph (i.e. the solution of the min–cut problem, see text).

defined for nodes and arcs (b_i and b_{ij}, respectively) as follows [2]:

$$b_i = \frac{1}{(N-1)(N-2)} \sum_{j,k \neq i} \frac{n_{jk}(i)}{n_{jk}} \tag{1}$$

$$b_{ij} = \frac{1}{N(N-1)} \sum_{k,l} \frac{n_{kl}(ij)}{n_{kl}} \tag{2}$$

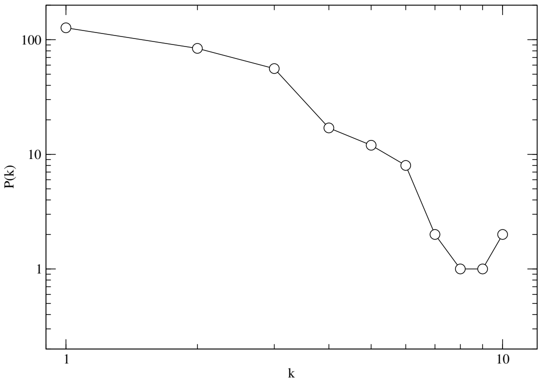

Fig. 2 – The distribution of HVIET node's degree (log–log scale).

They define the "relevance" of a given node (link) as being contained in a number of shortest path $n_{jk}(i)$ $(n_{kl}(ij))$ connecting all other nodes.

We have also evaluated the *information* centrality of the nodes, which relates a node importance to the ability of the network to respond to the removal of the node. If one defines the *efficiency* $E[G]$ of a graph as

$$E[G] = \frac{1}{N(N-1)} \sum_{i,j \in G} \frac{1}{d_{ij}} \tag{3}$$

where d_{ij} is the shortest–path distance between nodes i and j, then the *information* centrality of node i, I_i is

$$I_i = \frac{\Delta E}{E} = \frac{E[G] - E[G']}{E[G]} \tag{4}$$

where $E[G]$ and $E[G']$ are the *efficiency* of the unperturbed network and that of the network after the removal of node i, respectively. Table I provides the ranking of the nodes and arcs with respect to the values of the *betweenness* and the *information* centrality.

Each graph can be represented by an *Adjacency* matrix $\mathbf{A}$ whose elements $A_{ij} = 1$ if nodes i and j are connected (there is a link in between). Aside to the Adjacency matrix, one can define the *Laplacian* matrix $\mathbf{L}$ whose elements L_{ij} are defined as $L_{ij} = \delta_{ij} \sum_{j=1}^{N} A_{ij} - A_{ij}$. The eigenvector associated to its second eigenvalue has components of different signs; if one separates nodes whose eigenvector's component is positive from those with a negative eigenvector component, one can separate the network in two subsets, interacting each other with the `lowest number of links` (min–cut theorem) [8, 9]. This is a relevant property as it allows to define a *critical section*, that is an ideal cut line allowing to bisecate the network into the two largest connected subnetworks.

The application of the min–cut method to HVIET [3] yields the following bisection: the network is diveded into two, connected, parts HVIET_1 and HVIET_2, the first formed by $\mathrm{N(HVIET_1)}$=210, the second by $\mathrm{N(HVIET_2)}$=100 nodes. The two parts are separated by *only* four links; the removal of these links allows to totally bisecate the network, which would result to be separated into two, non-communicating, parts (see fig.1, right). The ideal line, joining the location of the removed links, could be called `first critical section`. This is a major outcome of the spectral analysys; this provides a way to locate the critical vulnerability lines of the network. This procedure can be iterated on the different components of the graphs, by creating `critical sections` of higher orders. The definition of critical sections of higher order could be a valuable solution for the problem of `islanding`, a procedure often used to isolate regions of the network to avoid cascade effects [10].

There are several nodes which are crucial from the topological standpoint: nodes 153, 154, 183 have the highest b_i and several links departing from these nodes have also high b_{ij} values. In terms of shortest–paths, these nodes and arcs belong to many shortest–paths connecting couples of nodes in the network. In topological terms, they are quite relevant; we will attempt to show if the electrical flow will make a "massive" use of these nodes or if the interplay between topology and technical properties of the different lines will introduce some modification to this scenario. Other than being highly central nodes, these nodes are also involved in the first critical section, as their removal allows to perform the maximal bisection. As such, they are also relevant as being points of structural vulnerability the network.

The DC power flow model. – We now introduce a flow model for the electrical power in the network which will allow to consider the "dynamical vulnerability" of the network.

Table I – *Rank of nodes on the basis of their degree k; nodes and lines with the highest betweenneess centrality b_i and b_{ij} (expressed in terms of the node's number and the origin–destination node's numbers, respectively). Highest ranked nodes for the information centrality I_i. Last column contains the four links resulting from the application of the min-cut theorem (see text; these links are those highlighted in fig.1 right–side). For these links, the ranking contained in the first column does not apply.*

rank	highest k (k)	highest b_i	highest b_{ij}	highest I_i	links of the first critical section
1	178(11)	183	153–168	201	142–145
2	158(11)	168	153–183	158	145–195
3	201(10)	154	183–207	183	154–212
4	211(8)	153	170–204	214	153–183
5	200(8)	207	119–170	103	

Dynamical vulnerability might be different from structural one: a link, in fact, which is topologically unrelevant, might have a central position in the electrical power transport.

We will use a simplified transport model as we wish to evaluate the effect of the network topology on the steady–state power flow rather than on transitory regimes. We will firstly evaluate the power flow distribution on the unperturbed network, resulting from a specific input–output condition, chosen to be representative of a typical power requirement that HVIET must daily sustain. Then we will perturb the network, by removing links, and we will detect if the "damaged" network is still able to produce a correct response to the input-output demand, within the imposed physical constraints.

We have modeled the transport of the electrical flow in the network by using a DC power flow model. The DC power flow equations [11] provide a linear relationship between the active power flowing through the lines and the power input into the nodes. They can be formulated as follows:

$$F_{km} = \frac{\theta_k - \theta_m}{x_{km}} \tag{5}$$

where x_{km} is the series reactance of the line connecting nodes k and m, F_{km} is the active power flow on this line and θ_k, θ_m are the voltage phase of the k'th and m'th node. Summing on all branches connected to the node k the power flow of that node P_k is

$$P_k = \sum_m F_{km} = \theta_k \sum_m x_{km}^{-1} - \sum_m \frac{\theta_m}{x_{km}} \tag{6}$$

this can be written in a matrix form as

$$\boldsymbol{P} = \boldsymbol{B\theta} \tag{7}$$

where $B_{km} = -1/x_{km}$ and $B_{kk} = \sum_l 1/x_{kl}$; $\boldsymbol{B}$ is a $N \times N$ matrix. Its rank is, however, $N-1$ since the network must comply with the conservation condition $\sum_{i=1}^N P_i = 0$. To solve the system, an equation is removed and the associated link is chosen in a way to introduce a reference node whose phase angle is arbitrarily set to $\theta = 0$. For a given input vector $\mathbf{P^0}$ the linear system in eq. (7) can be solved and a solution is found in terms of θ_{ij} and F_{ij}. Two constraints must be imposed to ensure the physical correctness of the solution. These comes from the fact that the DC power flow method results from the elimination of the imaginary part of the current equations, under the hypothesis that power phase angles are small. For this reason, it should result, $\forall(k, m)$, that

1. $\theta_{km} < 30$ degrees
2. $\mid F_{km} \mid < F_{km}^{max}$ (whereF_{km}^{max} is some specified limiting power flux on the link between nodes k and m).

If constraint (1) is not fulfilled, the inductive part of the electrical flux cannot be disregarded and eq.(7) does not hold. Constraint (2) is a technological threshold, relating to the specific line's impedance. Flux too large produce unendurable heat normally prevented by *ad hoc* elements which disconnect the line.

Data needed to solve the DC power flow system are thus: (a) the vector of input-output demand $P_i^{(0)}$ $(i = 1, N)$. Data used for the simulation represents a typical snapshot of the vector (injected power–extracted power) experienced daily in the HVIET. Source nodes are characterized by negative P_i values, junction by vanishing values, loads by positive values; (b) the line's reactance x_{ij}; (c) the values of the maximum sustainable power flux F_{ij}^{max} for each line.

The solution of the DC power flow system allows to evaluate: (1) the power flux F_{ij} along the lines resulting from the given input vector $\mathbf{P^{(0)}}$; (2) the phase angles θ_{ij}. These results will constitute the "normal" response of the network to the input conditions $\mathbf{P^{(0)}}$. The lines with the highest flux value are reported in Table II.

Perturbations and dynamical vulnerability. – Structural perturbations are imposed by removing an increasing number of links, and measuring the resulting distribution of the power flux as a function of the perturbation strength ξ, defined as the number of simultaneously–removed links. The more vulnerable parts of the networks will be those arcs whose removal will perturb the network at the point that it will not be able anymore to fulfill the dispatching of the intial power vector $\mathbf{P_{(0)}}$ (i.e. within the imposed constraints (1) and (2) above).

Starting from the input vector $\mathbf{P^{(0)}}$, we have carried out two series of simulations. In the first, we removed an increasing number of links: if the power flux established in the perturbed network is not consistent with the required constraints, the system is defined as $unsatisfied$ and its level of `Operability` (defined through a $Quality\ of\ Service$ (QoS) function) is set to zero. For a given perturbation strength, there will be cases in which the removal of specific lines produces a vanishing QoS, others in which, in turn, it produces a power flow redistribution still fulfilling the imposed contraints (in such a case, $QoS = 1$). In the cases where a part of the network (one single node or an entire subnetwork) is disconnected, the DC power flux equations cannot be solved and the dispatching problem cannot be fulfilled; also in this case $QoS = 0$. The value of the QoS associated to a given perturbation strength will be thus the average over the different values of the QoS issued upon the different choices of links removal ($< QoS >$ in fig.3 left).

All links have been separately removed ($\xi = 1$). We have thus removed all pairs of links ($\xi = 2$); for triplets ($\xi = 3$) and for quadruplets $\xi = 4$ of links, as an exaustive evaluation of all possible link's combinations cannot be attempted, we have evaluated an average over a large number of different cases.

In the second series of simulations, we repeat the same simulations with the difference that, when the perturbation is such to inhibit the system's satisfaction, we implement an optimization strategy which determines which is the optimal variation of the input vector $\mathbf{P}$ able to re–establish a correct power flow (i.e. within the imposed constraint). We call this procedure "re–dispatching". The "optimal" value is thus searched by reducing the input power (and the corresponding output power) in a way to maximize the function QoS

Table II – *Ranking of links according to their flux (relative to the* $\mathbf{P^{(0)}}$ *input) and the corresponding* F_{ij} *value. Last column contains the ranking of links according to the* ΔP *value (see text).*

rank	link (from node to node)	$F_{ij}(MW)$	link (from node to node)	ΔP (MW)
1	214–184	1326	214–184	1629
2	117–190	1310	127–103	1574
3	117–201	1281	127–186	1037
4	120–190	1102	117–190	938
5	127–103	990	117–165	872

$$QoS = 1 - \frac{\sum_{i \in loads}[P_i^{(0)} - P_i]}{\sum_{i \in loads} P_i^{(0)}} \tag{8}$$

where the value of P_i is the input vector resulting by the re–dispatching. Re–dispatching thus allows the fulfilment of the electrical problem, although with some degradation. The QoS measures the level of degradation that the network must suffer.

The results of the re–dispatching strategy can be used to establish a hierarchy of links, on the basis of the source reduction which is associated to their removal. In other words, when $\xi = 1$, each link is removed and the corresponding re–dispatching procedure is performed if the removal of that link produces the system's inoperability. If one relates the specific link with the associated value of $\Delta P = \sum_{i \in loads}[P_i(L_0) - P_i]$ one obtains the graph reported in fig.3 (right).

As we can see, there are a number of links whose removal introduces a severe perturbation to the network: although the network is still able to sustain a power flow, its global amount should be significantly reduced. There are a few links whose removal reduces the overall network's capacity of dispatching electrical power of an amount larger than 1500 MW: without

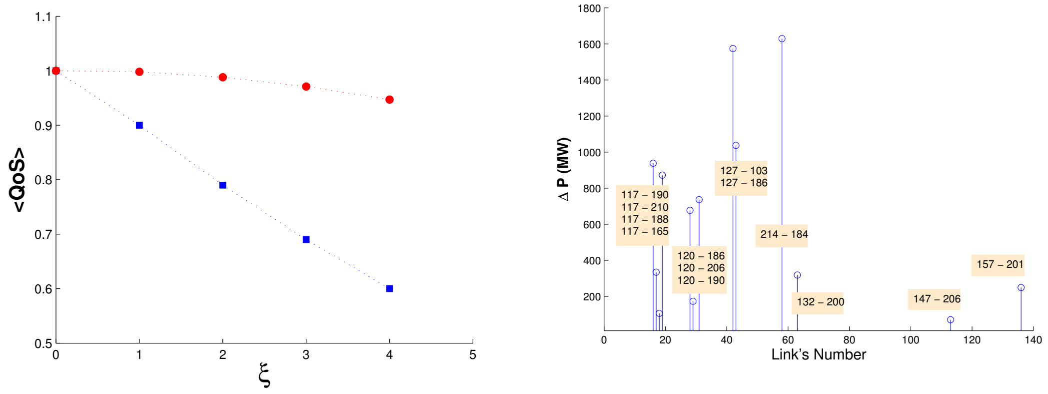

Fig. 3 – Left:QoS value as a function of the perturbation ξ. Full line represents the QoS values when no optimization is performed; dotted line shows the potential behavior of the network when the optimization procedure is adopted. Right: value of ΔP as a function of link's number. The nodes producing the need of a larger re–dispatching amount are indicated.

these lines, HVIET cannot sustain the flux of a total amount of power and dispacth it as required by the loads.

Table II shows that only a small number of nodes having a "functional" relevance for the network can be discovered through the topology analysis of its graph. The topological relevance of the "hub" nodes 201 and 214 allows them to be present both in the high–ranked k and I_i values and in the highest flux lines; apart from these, however, practically all other nodes which are highly fluxed and determine a high functional vulnerability (large value of ΔP) cannot be simply "predicted" by the use of the topology analysis alone.

Conclusions. – This work has shown how it is possible to set up "behavioral" models enabling to relate the network's topology to its level of function. A simple DC power flow method has been used to evaluate the "efficiency" of the network to sustain the flux of a given amount of electrical power, deduced by real data relative to the italian dispatching of high–voltage electrical power (380 kV). Topological analysis of the network has put in evidence a number of structural vulnerabilities, also deduced by the spectral analysis of the Laplacian matrix associated to the network's graph. The DC power flow simulation has allowed to establish a hierarchy of network's lines to which is associated a high functional vulnerability (the removal of these lines can be counterbalanced only by considerably reducing the power flow level which the network can sustain). It should be remarked, in conclusion, that topological analysis and the simulation of "behavioural" models (such as the DC power flow model) provide *complementary* informations which can be used to extract data concerning robustness and vulnerability properties of complex networks.

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